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THE SIGN OF THE DEVIL . . .
and the Sine of the Devil

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> *A certain wisdom is needed here; with a little ingenuity anyone can calculate the number of the beast, for it is a number that stands for a certain man. The man's number is six hundred sixty-six.*
>
> Revelation 13:18

That man, of course, is none other than Satan himself, he whose number is 666, the infamous "Sign of the Devil" or "Number of the Beast." Actually, 666 is a rather devilish number in and of itself, and it is connected in an intimate way to the classical golden ratio, also known as *phi*. In particular, I'll show that 666 can be expressed in terms of inverse trigonometric functions of phi. But lest we get ahead of ourselves, let's start by taking a quick look at the wonderful number phi.

Like its cousins π and e, the irrational number commonly denoted by the Greek letter phi (φ) often turns up in surprising situations. Phi was known as early as the times of the ancient Greeks, who thought of it as the ratio between the extreme and mean sections of a line segment (Figure 1). Here, the ratio **A/B** equals the ratio (**A** + **B**)/**A**, and both ratios equal φ, also known as the golden ratio or the divine proportion. The Greeks believed this ratio to be inherently pleasing, and many architects and artists incorporated it into their works. A notable example is the Parthenon, whose design uses so-called "golden-rectangles" – rectangles whose longer sides are in golden ratio to their shorter sides.

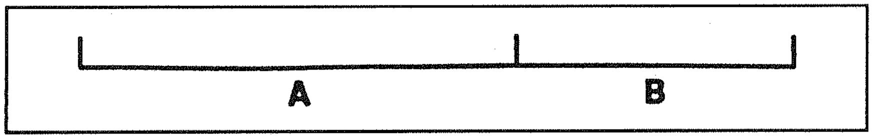

Figure 1.

Consider again the line segment divided into extreme and mean ratio. Recall that **A/B** = (**A** + **B**)/**A**. A little algebra gives us $(\mathbf{A/B})^2 - (\mathbf{A/B}) - 1 = 0$, which has the positive root

$$\mathbf{A/B} = (1 + \sqrt{5})/2 = \mathbf{1.618033988749...},$$

the number we call phi. These equations lead to some interesting numerical properties, such as $\varphi = 1/\varphi + 1$ and $\varphi^2 = \varphi + 1$. Continued fractions and square roots give us additional ways of representing phi:

$$\varphi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \dots}}} \quad \text{and} \quad \varphi = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}}$$

In the 18th century, phi was discovered within the Fibonacci sequence, a sequence that appears unexpectedly in many organic and physical phenomena. Much has been written on its applications in topics as diverse as phyllotaxis (the arrangement of leaves on a stem), electrical circuits, reflection of light rays, the arrangement of florets on sunflowers, and the arrangement of scales on fir cones and pineapples. The sequence first appeared in the answer to a problem posed by the mathematician Leonardo Pisano, also known as "Fibonacci," in his 1202 work *Liber Abaci*: Say we have a pair of breeding rabbits that start to bear young two months after they are born, bearing only a single male/female pair at the end of each month. If each of these pairs bears young similarly, and none dies, how many pairs will there be after one year? It turns out that the number of pairs after each month forms a sequence in which each term is the sum of the two terms before it:

1, 2, 3, 5, 8, 13, . . . ,

with a total of 377 pairs after one year. This sequence is a part of what has become known as the Fibonacci sequence:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ...

Using this sequence, we can approximate phi by taking the ratio of any term to the term preceding it. These ratios, which are alternately greater than and less than phi, approach phi as the number of terms goes to infinity. For instance, to four places, 8/5 = 1.6000, 13/8 = 1.6250, 21/13 = 1.6153, 34/21 = 1.6190, 55/34 =

1.6176, and so on. Phi's relationship to the Fibonacci sequence is also revealed in Binet's formula for the *n*th term of the sequence, F_n:

$$F_n = \frac{1}{\sqrt{5}} \left[\left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right].$$

Let me introduce another property of phi, an identity relating phi with the satanic number 666. I discovered this identity one day in May 1988 while idly punching numbers into my calculator. I happened to enter 666, and then I chanced to hit the *sine* key, which returned –0.809016... . This number seemed oddly familiar, but I couldn't figure out where I might have seen it. A few days later, I felt a revelation. I keyed in *sin*(666) again, and I multiplied the result by various constants, eventually trying –2. The answer was 1.6180339... , and I then knew I had it: phi equals –2 times the sine of 666!

Using a computer, a friend and I confirmed this result numerically to fourteen digits – the maximum precision of our computer. But how could I prove it formally? I tried to prove the identity using Taylor series, but I gave up after a month of getting nowhere. But a few months later, on August 30, 1988, I stumbled upon a diagram of a golden triangle, and the answer revealed itself. The proof was waiting in simple geometry.

I quote the following from "Golden Triangles, Rectangles and Cuboids" by Marjorie Bicknell and Verner E. Hoggatt, Jr.:

> Consider the isosceles triangle with a vertex angle of 36°. On bisecting the base angle of 72°, two isosceles triangles are formed, and ΔBDC is similar to ΔABC as indicated in (Figure 2).
>
> Since ΔABC is similar to ΔBDC, (AB/BD) = (BC/DC), or $y/x = x/(y-x)$, so that $y^2 - yx - x^2 = 0$. Dividing through by $x^2 \neq 0$, $(y^2/x^2) - (y/x) - 1 = 0$. The quadratic equation gives $(y/x) = (1 + \sqrt{5})/2 = \varphi$ as the positive root, so ΔABC is a Golden Triangle.

To this diagram we add a segment ED bisecting ∠ADB. Since ΔABD is isosceles, segment ED is perpendicular to segment AB (Figure 3).

We know that *sin*(∠ADE) = AE/AD. Multiplying numerator and denominator by 2 gives 2(AE)/2(AD) = AB/2(AD), which equals $y/2x = \varphi/2$ in the first figure. Since ∠ADE measures 54°, *sin*(–54°) = – φ/2. Now we add two revolutions, or 720°, to the angle, and we get *sin*666° = – φ/2, or 666° = *arcsin*(– φ/2) + 720°. ***Q.E.D.***

I showed these findings to my freshman calculus teacher, Dr. Robert Connelly, Professor of Mathematics at Cornell University. He promptly discovered a similarly suspicious identity: *cos*[(6)(6)(6)°] = – φ/2, which can be proved in the same manner. Combining these two identities, we find

$$\varphi = -\{\, \sin 666^\circ + \cos[(6)(6)(6)^\circ] \,\}.$$

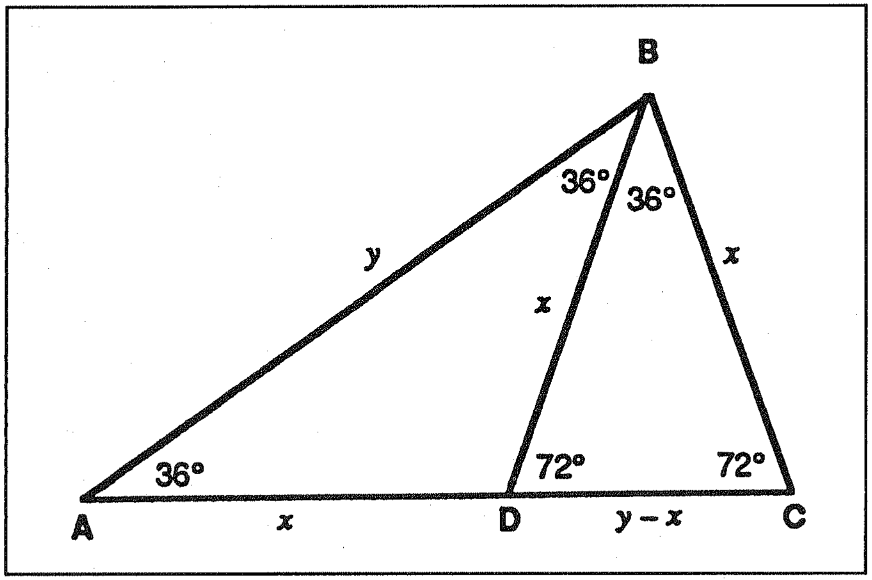

Figure 2.

Hence, phi, the divine proportion of classical art, is a sum of trigonometric functions of satanic numbers.

So then, what does all this mean? Could it be that the ubiquitous phi is really a creation of the Devil? Keep in mind that the pentagram – a five-pointed star with every segment in golden ratio to the next smallest segment – has long been associated with demonic cults. Does this mean that mathematics is the work of Satan, and therefore mathematicians are Satanists? I haven't the faintest idea, but I'm definitely keeping my mouth shut, and I'd advise all of you to do the same. We wouldn't want this information to get into the wrong hands.

Postscript

There are, of course, many other ways of proving this identity. In an algebra class during my senior year at Cornell, one of our problem sets asked us to prove it using properties of complex numbers. I asked the professor where he had found the identity, and he told me that he had heard it from a colleague in the math department, whose student had discovered it. He was surprised to learn that I was that student. When he introduced the problem set in class, he noted that one of the problems asked us to prove an interesting identity, and "just to show that it really is a small world, the student who discovered the identity is here with us in the classroom!"

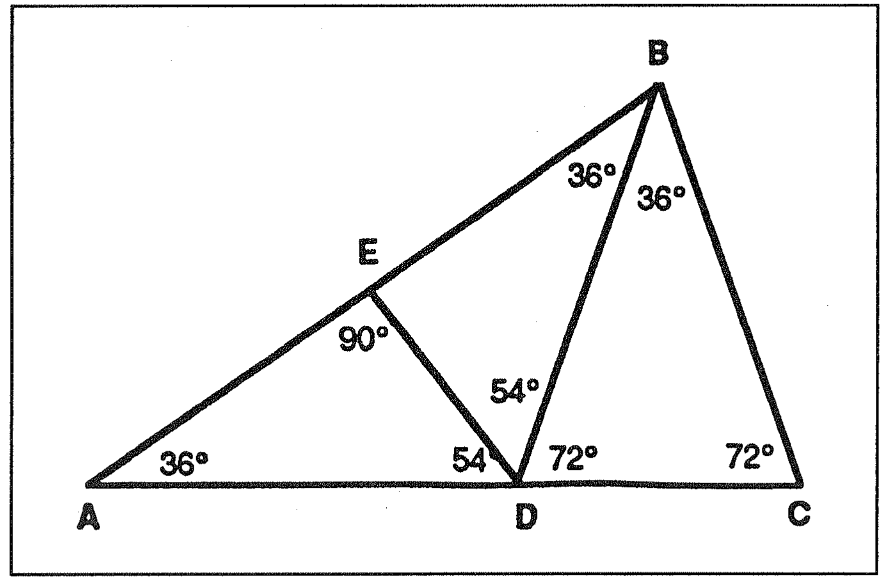

Figure 3.

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